22137304

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

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Friday 10 May 2013 (morning)

1 hour 30 minutes

Examination code

2	2	1	3	–	7	3	0	4

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 7]*

An arithmetic sequence is given by $5, 8, 11, \ldots$.

(a) Write down the value of d. *[1 mark]*

(b) Find

(i) u_{100};

(ii) S_{100}. *[4 marks]*

(c) Given that $u_n = 1502$, find the value of n. *[2 marks]*

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2. *[Maximum mark: 6]*

Consider the following cumulative frequency table.

x	Frequency	Cumulative frequency
5	2	2
15	10	12
25	14	26
35	p	35
45	6	41

(a) Find the value of p. *[2 marks]*

(b) Find

(i) the mean;

(ii) the variance. *[4 marks]*

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3. *[Maximum mark: 5]*

In the expansion of $(3x-2)^{12}$, the term in x^5 can be expressed as $\binom{12}{r}\times(3x)^p\times(-2)^q$.

(a) Write down the value of p, of q and of r. *[3 marks]*

(b) Find the coefficient of the term in x^5. *[2 marks]*

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4. *[Maximum mark: 6]*

Consider the system of equations $\begin{aligned} -x - y + z &= 2.5 \\ x + y &= 1 \\ -2x - y + 2z &= -3 \end{aligned}$

This system can be represented by the matrix equation $\boldsymbol{AX} = \boldsymbol{B}$, where $\boldsymbol{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

(a) (i) Write down the matrix $\boldsymbol{A}$.

(ii) Write down the matrix $\boldsymbol{A}^{-1}$. *[3 marks]*

(b) Hence, find $\boldsymbol{X}$. *[3 marks]*

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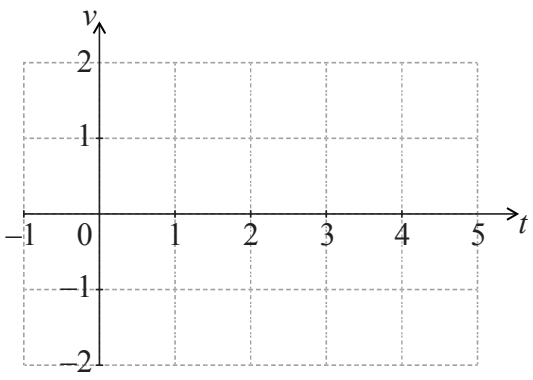
5. *[Maximum mark: 8]*

The velocity of a particle in ms^{-1} is given by $v = e^{\sin t} - 1$, for $0 \le t \le 5$.

(a) On the grid below, sketch the graph of v. *[3 marks]*



(b) (i) Write down the positive t-intercept.

(ii) Find the total distance travelled by the particle in the first five seconds. *[5 marks]*

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6. *[Maximum mark: 6]*

Let f and g be functions such that $g(x) = 2f(x+1) + 5$.

(a) The graph of f is mapped to the graph of g under the following transformations:

vertical stretch by a factor of k, followed by a translation $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

(i) k;

(ii) p;

(iii) q. *[3 marks]*

(b) Let $h(x) = -g(3x)$. The point $\mathrm{A}(6, 5)$ on the graph of g is mapped to the point A' on the graph of h. Find A'. *[3 marks]*

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7. *[Maximum mark: 7]*

A random variable X is normally distributed with $\mu = 150$ and $\sigma = 10$.

Find the interquartile range of X.

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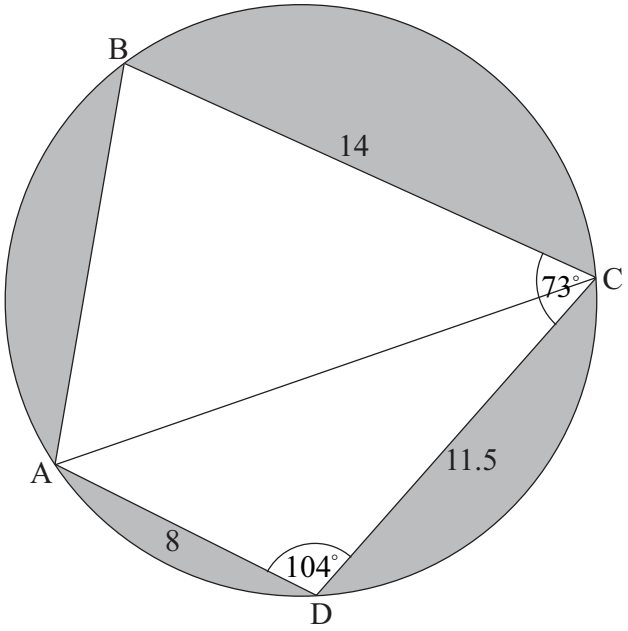
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

The diagram shows a circle of radius 8 metres. The points ABCD lie on the circumference of the circle.



BC = 14 m, CD = 11.5 m, AD = 8 m, $A\hat{D}C = 104^\circ$, and $B\hat{C}D = 73^\circ$

(a) Find AC. *[3 marks]*

(b) (i) Find $A\hat{C}D$.

(ii) Hence, find $A\hat{C}B$. *[5 marks]*

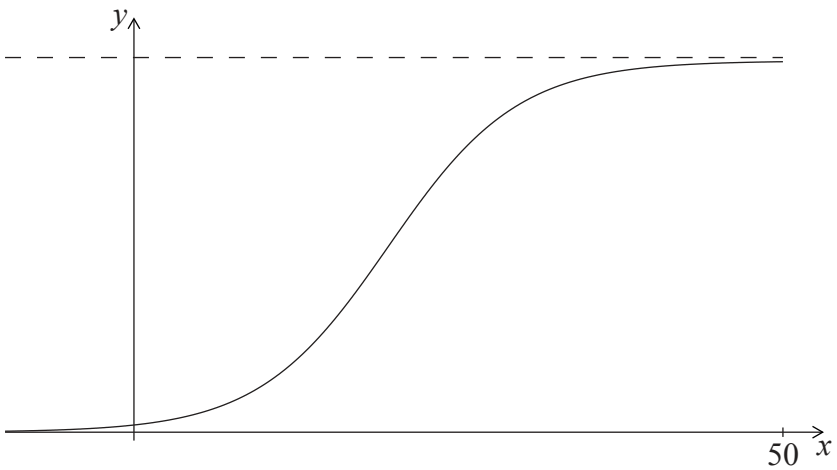
(c) Find the area of triangle ADC. *[2 marks]*

(d) Hence or otherwise, find the total area of the shaded regions. *[4 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

Let $f(x) = \frac{100}{\left(1 + 50e^{-0.2x}\right)}$. Part of the graph of f is shown below.



(a) Write down $f(0)$. *[1 mark]*

(b) Solve $f(x) = 95$. *[2 marks]*

(c) Find the range of f. *[3 marks]*

(d) Show that $f'(x) = \frac{1000e^{-0.2x}}{\left(1 + 50e^{-0.2x}\right)^2}$. *[5 marks]*

(e) Find the maximum rate of change of f. *[4 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 16]*

A Ferris wheel with diameter 122 metres rotates clockwise at a constant speed. The wheel completes 2.4 rotations every hour. The bottom of the wheel is 13 metres above the ground.

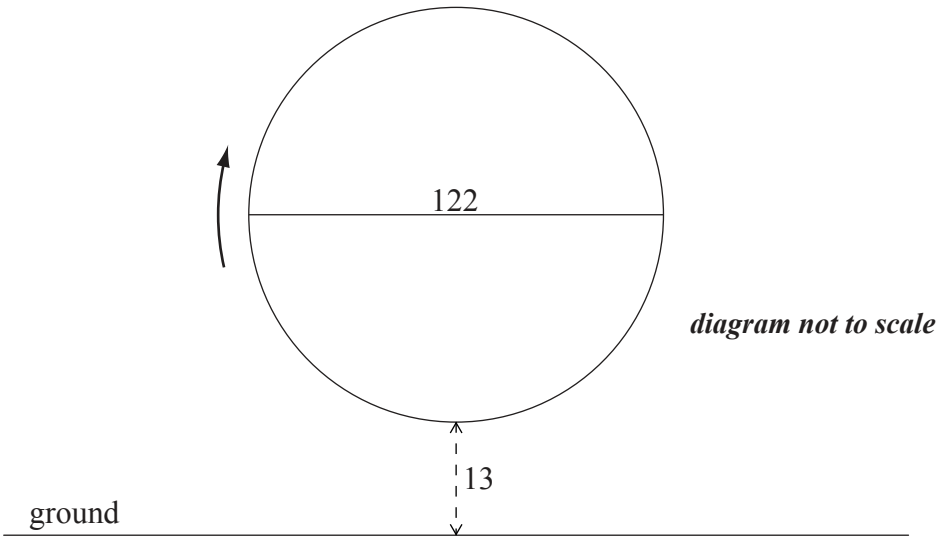


diagram not to scale

A seat starts at the bottom of the wheel.

(a) Find the maximum height above the ground of the seat. *[2 marks]*

After t minutes, the height h metres above the ground of the seat is given by

$$h = 74 + a\cos bt\ .$$

(b) (i) Show that the period of h is 25 minutes.

(ii) Write down the **exact** value of b. *[2 marks]*

(c) Find the value of a. *[3 marks]*

(d) Sketch the graph of h, for $0 \le t \le 50$. *[4 marks]*

(e) In one rotation of the wheel, find the probability that a randomly selected seat is at least 105 metres above the ground. *[5 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.